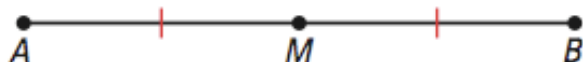


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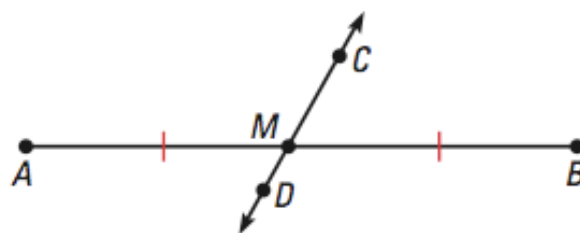
- I can define a midpoint and bisector.
- I can use midpoints and bisectors to solve problems.

The **midpoint** of a segment is the point that divides, or **bisects**, the segment into two congruent segments. In this book, matching red *congruence marks* identify congruent segments in diagrams.

A **segment bisector** is a segment, ray, line, or plane that intersects a segment at its midpoint.



M is the midpoint of \overline{AB} if
 M is on \overline{AB} and $AM = MB$.

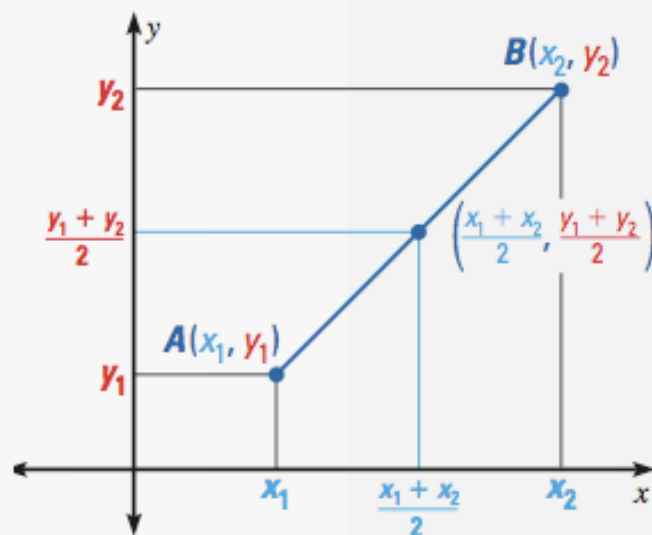


\overleftrightarrow{CD} is a bisector of \overline{AB} .

THE MIDPOINT FORMULA

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint of \overline{AB} has coordinates

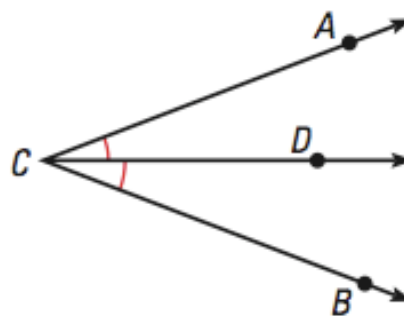
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$



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An **angle bisector** is a ray that divides an angle into two adjacent angles that are congruent. In the diagram at the right, the ray \overrightarrow{CD} bisects $\angle ABC$ because it divides the angle into two congruent angles, $\angle ACD$ and $\angle BCD$.

In this book, matching *congruence arcs* identify congruent angles in diagrams.



$$m\angle ACD = m\angle BCD$$

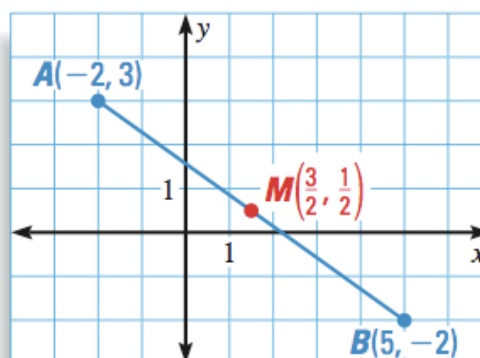
EXAMPLE 1 Finding the Coordinates of the Midpoint of a Segment

Find the coordinates of the midpoint of \overline{AB} with endpoints $A(-2, 3)$ and $B(5, -2)$.

SOLUTION

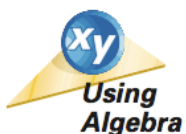
Use the Midpoint Formula as follows.

$$\begin{aligned} M &= \left(\frac{-2 + 5}{2}, \frac{3 + (-2)}{2} \right) \\ &= \left(\frac{3}{2}, \frac{1}{2} \right) \end{aligned}$$



1. Find the midpoint of \overline{DE} with endpoints $D(3, 5)$ and $E(-4, 0)$.
2. Find the midpoint of \overline{XY} with endpoints $X(3, -4)$ and $Y(-3, -1)$.

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EXAMPLE 2 Finding the Coordinates of an Endpoint of a Segment

The midpoint of \overline{RP} is $M(2, 4)$. One endpoint is $R(-1, 7)$. Find the coordinates of the other endpoint.

SOLUTION

Let (x, y) be the coordinates of P .
Use the Midpoint Formula to write equations involving x and y .

$$\frac{-1 + x}{2} = 2$$

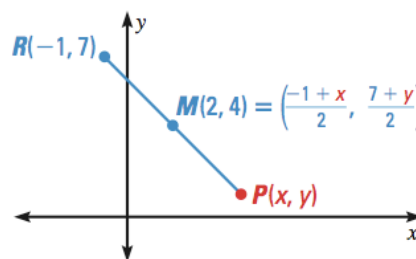
$$-1 + x = 4$$

$$x = 5$$

$$\frac{7 + y}{2} = 4$$

$$7 + y = 8$$

$$y = 1$$



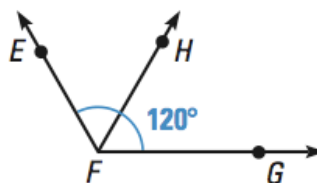
► So, the other endpoint of the segment is $P(5, 1)$.

3. The midpoint of \overline{XY} is $M(3, -4)$. One endpoint is $Y(-3, -1)$. Find the coordinates of the other endpoint.

4. The midpoint of \overline{JK} is $M\left(0, \frac{1}{2}\right)$. One endpoint is $J(2, -2)$. Find the coordinates of the other endpoint.

EXAMPLE 3 *Dividing an Angle Measure in Half*

The ray \overrightarrow{FH} bisects the angle $\angle EFG$.
Given that $m\angle EFG = 120^\circ$, what are the measures of $\angle EFH$ and $\angle HFG$?



SOLUTION

An angle bisector divides an angle into two congruent angles, each of which has half the measure of the original angle. So,

$$m\angle EFH = m\angle HFG = \frac{120^\circ}{2} = 60^\circ.$$

5. \overrightarrow{JK} bisects $\angle HJL$. Given that $m\angle HJL = 42^\circ$, what are the measures of $\angle HJK$ & $\angle KJL$?

EXAMPLE 4 *Doubling an Angle Measure*

KITE DESIGN In the kite, two angles are bisected.

$\angle EKI$ is bisected by \overrightarrow{KT} .

$\angle ITE$ is bisected by \overrightarrow{TK} .

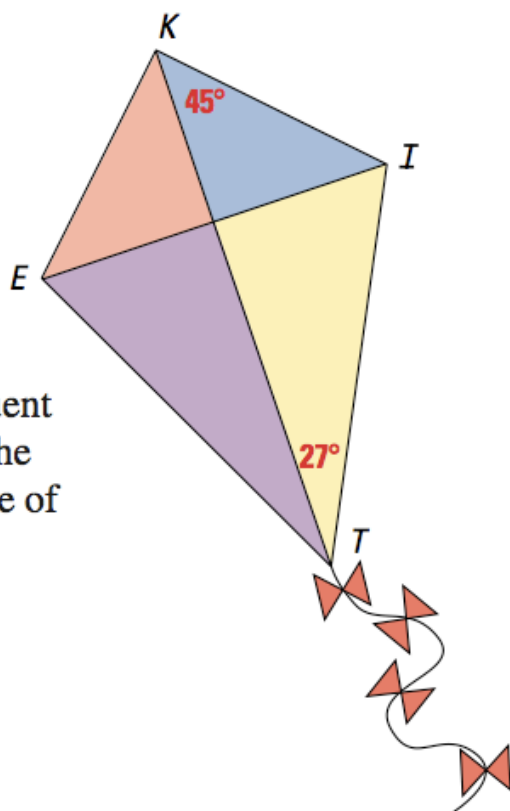
Find the measures of the two angles.

SOLUTION

You are given the measure of one of the two congruent angles that make up the larger angle. You can find the measure of the larger angle by doubling the measure of the smaller angle.

$$m\angle EKI = 2m\angle TKI = 2(45^\circ) = 90^\circ$$

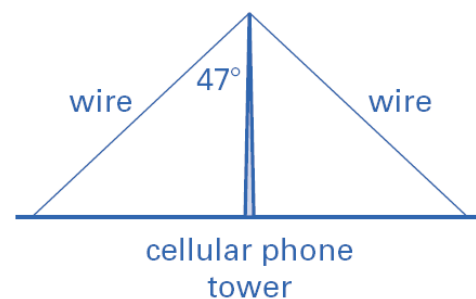
$$m\angle ITE = 2m\angle KTI = 2(27^\circ) = 54^\circ$$



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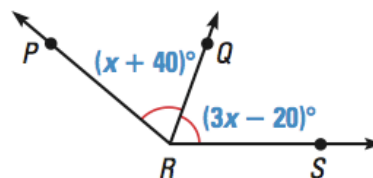
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6. A cellular phone tower bisects the angle formed by the two wires that support it. Find the measure of the angle formed by the two wires.



EXAMPLE 5 Finding the Measure of an Angle

In the diagram, \overrightarrow{RQ} bisects $\angle PRS$. The measures of the two congruent angles are $(x + 40)^\circ$ and $(3x - 20)^\circ$. Solve for x .



SOLUTION

$$m\angle PRQ = m\angle QRS$$

$$(x + 40)^\circ = (3x - 20)^\circ$$

$$x + 60 = 3x$$

$$60 = 2x$$

$$30 = x$$

Congruent angles have equal measures.

Substitute given measures.

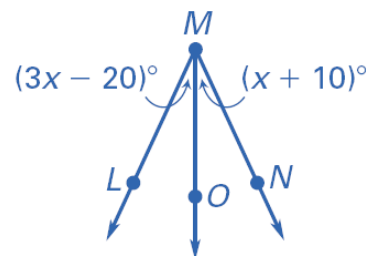
Add 20° to each side.

Subtract x from each side.

Divide each side by 2.

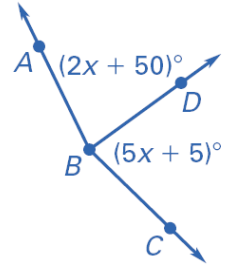
- So, $x = 30$. You can check by substituting to see that each of the congruent angles has a measure of 70° .

7. In the diagram \overline{MO} bisects $\angle LMN$. The measures of the two congruent angles are $(3x - 20)^\circ$ & $(x + 10)^\circ$. Solve for x .



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8. In the diagram \overline{BD} bisects $\angle ABC$. Find x and use it to find $m\angle ABD$, $m\angle DBC$, & $m\angle ABC$.



9. What kind of geometric figure is an *angle bisector*?
10. How do you indicate congruent segments in a diagram? How do you indicate congruent angles in a diagram?
11. What is the simplified form of the Midpoint Formula if one of the endpoints of a segment is $(0, 0)$ and the other is (x, y) ?

Weapons of Math Destruction™

